METHOD AND APPARATUS FOR SOLVING SYSTEMS OF NONLINEAR EQUATIONS USING INTERVAL ARITHMETIC

ABSTRACT

One embodiment of the present invention provides a computer-based system for solving a system of nonlinear equations specified by a vector function, **f**, wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, wherein **x** is a vector $(x_1, x_2, x_3, \ldots x_n)$. The system operates by receiving a representation of a subbox $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, wherein for each dimension, i, the representation of X_i includes a first floating-point number, a_i, representing the left endpoint of X_i, and a second floating-point number, b_i, representing the right endpoint of X_i. The system stores the representation in a computer memory. Next, the system applies term consistency to the set of nonlinear equations, $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, over **X**, and excludes portions of **X** that violate the set of nonlinear equations. The system also applies box consistency to the set of nonlinear equations over **X**, and excludes portions of **X** that violate the set of nonlinear equations. Finally, the system performs an interval Newton step on **X** to produce a resulting subbox **Y**, wherein the point of expansion of the interval Newton step is a point **x** within **X**, and wherein performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using interval arithmetic to produce an interval result $\mathbf{f}^I(\mathbf{x})$. The system integrates the sub-parts of the process with branch tests designed to increase the overall speed of the process.